

April 2016 Problem of the Month

Tangents

Given a

$$\prod_{k=1}^n (1 + a_k) = 2^n;$$

find (a_1, a_2, \dots, a_n) .

Solution

For a

$$\begin{aligned} 1 + a_k &= 1 + \frac{k}{k} = \frac{k + k}{k} \\ &= \frac{\sqrt{2} (45 + k)}{k} = \frac{\sqrt{2} k}{k (45 - k)}. \end{aligned}$$

Then

$$(1 + a_k)(1 + a_{(45 - k)}) = \frac{\sqrt{2} (45 - k)}{k} \cdot \frac{\sqrt{2} k}{(45 - k)} = 2;$$

It follows that

$$\begin{aligned} \prod_{k=1}^n (1 + a_k) &= (1 + a_1)(1 + a_2) \cdots (1 + a_n) \\ &= 2 \cdot 2 \cdots 2 = 2^{23}; \end{aligned}$$

$n = 23$.