April 2016 Problem of the Month

Tangents

G a

$$\prod_{k=1}^{45} (1 + \tan k^\circ) = 2^n;$$

fi (a) a n: (k a a a k a .)

Solution

F a

$$1 + \tan k^\circ = 1 + \frac{\sin k^\circ}{\cos k^\circ} = \frac{\cos k^\circ + \sin k^\circ}{\cos k^\circ}$$
$$= \frac{\sqrt{2}\sin(45^\circ + k^\circ)}{\cos k^\circ} = \frac{\sqrt{2}\sin(45^\circ - k^\circ)}{\cos k^\circ}:$$

T

$$(1 + \tan k^\circ)(1 + \tan(45^\circ - k^\circ)) = \frac{\sqrt{2}\sin(45^\circ - k^\circ)}{\cos k^\circ} \cdot \frac{\sqrt{2}\cos k^\circ}{\sin(45^\circ - k^\circ)} = 2:$$

I a

$$\prod_{k=1}^{45} (1 + \tan k^\circ) = (1 + \tan 45^\circ) \prod_{k=1}^{22} (1 + \tan k^\circ)(1 + \tan(45^\circ - k^\circ))$$
$$= 2 \prod_{k=1}^{22} 2 = 2^{23};$$

$n = 23$: