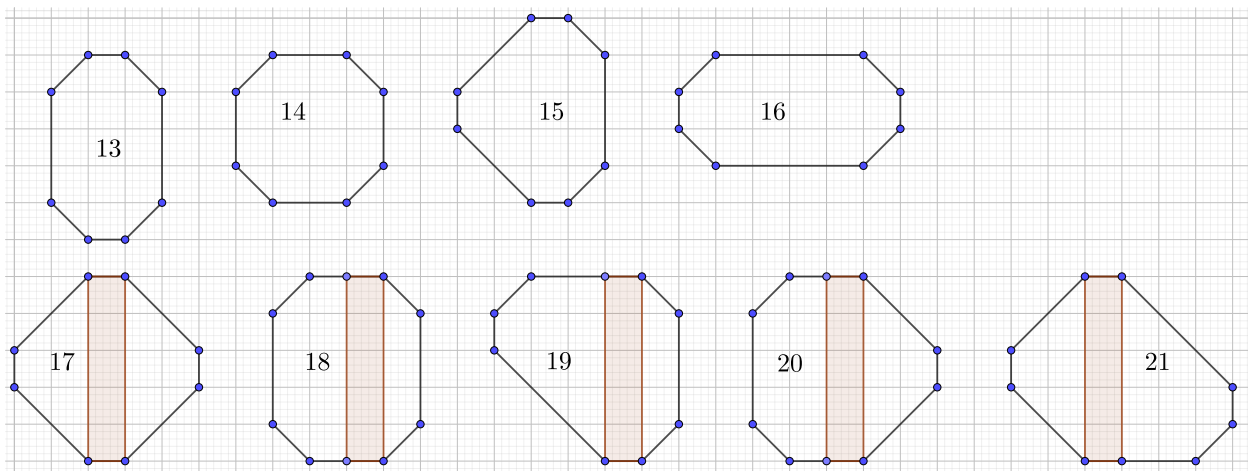


PROBLEM OF THE MONTH, OCTOBER 2019, SOLUTION

An octagon is said to be *nice* if all its interior angles are 135° , its vertices are points with integer coordinates, and its area is a positive integer. Prove that for every positive integer $A \geq 13$, there exists a nice octagon whose area is $= A$.

Solution

Here are the diagrams of nice octagons of areas 13 through 21.



Note that for the last five (17–21) there is a shaded rectangle within each octagon exactly 5 units tall and one unit wide. To find a nice octagon of area 22, one would only need to widen the column in the octagon of area 17 by one unit. This would create the extra 5 square units. Since we have five consecutive nice octagons with this feature, it is clear that by expanding the width of these columns we can eventually achieve any area.