PROBLEM OF THE MONTH, NOVEMBER 2017

Let $M(0;1;2)$ be the set of rectangular matrices whose entries are 0, 1, or 2.

A matrix in $M(0;1;2)$ is said to be *defective* if there exists a 2 2 submatrix all whose entries are equal to each other. A matrix with no defect is called *perfect*.

For example the 7 7 matrix A below is defective since the 2 2 submatrix created by rows 4 and 6 and columns 1 and 7 has all the entries equal to 1. On the other hand, the 7 7 matrix B is perfect.

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 2 & 1 & 2 \\ 0 & 1 & 2 & 1 & 0 & 0 & 0 \\ 1 & 0 & 2 & 2 & 0 & 1 & 2 \\ 1 & 1 & 0 & 2 & 2 & 0 & 1 \\ 2 & 1 & 1 & 0 & 2 & 2 & 0 \\ 1 & 2 & 2 & 0 & 1 & 0 & 1 \\ 1 & 2 & 1 & 0 & 0 & 2 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 0 & 0 & 0 & 2 & 1 & 2 \\ 0 & 1 & 2 & 1 & 0 & 0 & 0 \\ 1 & 0 & 2 & 2 & 0 & 1 & 2 \\ 1 & 1 & 0 & 2 & 2 & 0 & 1 \\ 2 & 1 & 1 & 0 & 2 & 2 & 0 \\ 2 & 2 & 2 & 0 & 1 & 0 & 1 \\ 1 & 2 & 1 & 0 & 0 & 2 & 2 \end{bmatrix}$$

Here is the problem: *nd a* 10 10 *perfect matrix in* $M(0;1;2)$.

Submit your solutions to professor Dan Ismailescu, Mathematics Department via email at dan.p.ismailescu@hofstra.edu, or bring it in person at 103C Roosevelt Hall.