SOLUTION - PROBLEM OF THE MONTH, FEBRUARY 2018

Congratulations to Jason Belanger, Mayank Jain and Piotr Laskawiec for submitting correct solutions to the February problem!

Problem. Now that 2017 just ended and 2018 just began, here is a nice problem for you to think about.

Find a positive integer n such that the first four digits of n^3 are 2018 and the last four digits of n^3 are 2017. In other words, n^3 has to have the following form

$$n^3 = 2018 \ldots 2017$$

For additional bragging rights, prove that there exist infinitely many such positive integers.

Submit your solutions to professor Dan Ismailescu, Mathematics Department via email at dan.p.ismailescu@hofstra.edu, or bring it in person at 103C Roosevelt Hall.

Solution

For x^3 to end in 2017 one needs that $x \equiv 9073 \pmod{10000}$. On the other hand, it is not hard to find numbers whose cubes start with 2018. Two such examples are $5866^3 = 201848801896$ or $27228^3 = 20185858556352$. Numbers that have both properties can then be constructed by combining two such numbers. Here are such examples:

586609073

5866009073

58660009073

586600⋯009073

272289073

2722809073

27228009073

2722800⋯009073